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ON DEVELOPMENTS OF FIXED POINT RESULTS IN DISLOCATED METRIC SPACE

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Abstract

The theory of fixed point is a very extensive field, which has numerous applications in various fields. The present paper deals with some developments on Fixed Point Results in Dislocated Metric Space which have further remarkable generalizations and extensions under several generalized metric spaces.

Key Words : *Common fixed point, Contraction, Dislocated metric space, Compatible mapp-pings, Self-mappings.*

Mathematics Subject Classification(MSC) : 47H10, 54H25

1. Introduction

Fixed point theory is an important area of analysis which is applicable in different fields. If T is a self mapping of a metric space (X, d) then, a point x in X is said to be fixed point of T if $Tx = x$; that is, a point which remains invariant under a self mapping is called a fixed point [36]. The concept of fixed point was initiated by Poincare, H. in 1886 [16]. Similarly, the concept of metric space was introduced by Frechet, M. in 1906 which furnished the common idealization of a large number of mathematical, physical and other scientific construction in which the notion of distance appears [16]. The first fixed point theorem is due to Brouwer, L.E.J. in 1912 [16]. In 1922, S. Banach proved a fixed point theorem for contraction mapping in metric space [3]. Since then, a number of fixed point theorems have been proved by different authors and many generalizations of this notion have been established. Von, J. in 1932, pointed out how fixed point theory could be utilized to prove the existence of equilibrium in economic models in a seminar on the topic on a system of economic equations and a generalization of Brouwer's fixed point theorem. This concept has lead to win Nobel Prizes in economics for Arrow, K. in 1972 and Debreu, G. in 1983. Applications of fixed point to Game theory also led to win a Nobel Prize in Economics for Nash, J. in 1994. In 1986, Matthews, S.J. introduced some concept of the notion of dislocated metric in the context of metric domains in his Ph.D. thesis[22]. In 2000, Hitzler, P. and Seda, A. K. generalized the notion of topology by relaxing the requirement that neighborhoods of a point includes the point itself and by allowing neighborhoods of points to be empty so that self distance of a point need not be equal to zero[9].

This paper deals with the historical developments of fixed point results in dislocated metric space. During the recent years, a number of fixed point results have been established by different authors for single and pair of mappings in dislocated metric spaces.The study of common fixed points of mappings in dislocated metric space satisfying certain contractive conditions has been the center of vigorous research activities.

2. Preliminaries

We start with the following definitions, propositions, lemmas, theorems and examples which are described in the main work.

Definition 1 [9] : Let X be a non empty set and let $d : X \times X \to [0, \infty)$ be a function satisfying the following conditions:

1. $d(x, y) = d(y, x)$,
2. $d(x, y) = d(y, x) = 0$ implies $x = y$ and
3. $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$.

Then d is called dislocated metric(or simply d-metric) on X.

Definition 2 [9] : A sequence $\{x_n\}$ in a dislocated metric space (X, d) is called a Cauchy sequence if for given $\epsilon > 0$, there corresponds $n_0 \in N$ such that for all $m, n \geq n_0$, we have $d(x_m, x_n) < \epsilon$.

Definition 3 [9] : A sequence in dislocated metric space converges with respect to d (or in d) if there exists $x \in X$ such that $d(x_n, x) \to 0$ as $n \to \infty$.

In this case, x is called limit of $\{x_n\}$ (or in d) and we write $x_n \to x$.

Definition 4 [9] : A dislocated metric space (X, d) is called complete if every Cauchy sequence in it is convergent with respect to d

Definition 5 [9] : Let (X, d) be a dislocated metric space. A mapping $T : X \to X$ is called contraction if there exists a number λ with $0 \leq \lambda < 1$ such that $d(Tx, Ty) \leq \lambda d(x; y)$.

Also, Hitzler, P. and Seda, A. K. included the following lemmas without proofs.

Theorem 1 [9] : Let (X, d) be a complete dislocated metric space. Let $T : X \to X$ be a continuous mapping satisfying $d(Tx, Ty) \leq \lambda d(x, y); 0 \leq \lambda < 1$. Then, T has a unique fixed point.

One more important definition has also been included over there which is as follows:

Definition 6 [9] : Let T and S be mappings from a metric space (X, d) into itself. Then T and S are said to be weakly compatible if they commute at their coincidence point, that is, $Tx = Sx$ for some $x \in X \Rightarrow TSx = STx$.

In 2002, Aamri, D. and Moutawakil, D.E., established "Some new common fixed point theorems under strict contractive conditions" [2]. The aim of this paper was to define a new property which generalizes the concept of noncompatible mappings and gives some common fixed point thorems under strict contractive conditions.

It is well known that in the setting of metric space, strict conractive condition does not ensure the existence of common fixed point unless the space is assumed compact or the strict conditions are replaced by stronger conditions as given by Jachymski, J. [12], Jungck, G. et.al. [17] and Pant, R.P. [28]. In 1986, Jungck, G. [18] introduced the notion of compatible maps. This concept was frequently used to prove existence theorems in common fixed point theory. However, the study in common fixed points of noncompatible mappings is also very interesting. They begin with some known definitions.

The notion of the weak commutativity introduced by Sessa, S. [35] is as follows:

Definition 7 [35] : Let A and S be two self mappings on a set X. Mappings A and S are said to be commuting if $ASx = SAx \forall x \in X$.
Definition 8 [35] : Let A and S be two self mappings on a set X. If $Ax = Sx$ for some $x \in X$, then x is called coincidence point of A and S.
Definition 9 [35] : Two selfmappings S and T of a metric space (X, d) are said to be weakly commuting pair if

$$d(STx, TSx) \leq d(Sx, Tx), \forall x \in X.$$

It is clear that two commuting mappings are weakly commuting but the converse is not true as shown by Sessa, S. [35]. Jungck, G.[18] extended this concept at first in the following way along with the foiiowing definitions.
Definition 10 [18] : Let T and S be two selfmapping of a metric space (X, d). Then S and T are said to be compatible if $\lim\limits_{n\to\infty} d(STx_n, TSx_n) = 0$ whenever (x_n) is a sequence in X such that $\lim\limits_{n\to\infty} Sx_n = \lim\limits_{n\to\infty} Tx_n = t$ for some $t \in X$.
Obviously, commuting pairs are weakly commuting but the converse is false as shown as below [18].
Example 1 [18] : Let $X = [0, 1]$ with the usual metric. Define $f(x) = \frac{x}{2}$ and $g(x) = \frac{x}{2+x}$

$$fgx = f[g(x)] = f\left(\frac{x}{2+x}\right) = \frac{\frac{x}{2+x}}{2} = \frac{x}{4+2x}$$

and

$$gfx = g[f(x)] = g\left(\frac{x}{2}\right) = \frac{\frac{x}{2}}{2+\frac{x}{2}} = \frac{x}{4+x}.$$

Then, for all x in X, one obtains

$$d(fgx, gfx) = \frac{x}{4+x} - \frac{x}{4+2x} = \frac{x^2}{(4+x)(4+2x)} \leq \frac{x^2}{4+2x} = \frac{x}{2} - \frac{x}{2+x} = d(fx, gx)$$

and f and g commute weakly. But, for any non-zero $x \in X$, we have

$$gfx = \frac{x}{4+x} > \frac{x}{4+2x} = fgx$$

and f and g do not commute.
Recently, Jungck, G. [18] have introduced the concept of weakly compatible maps as follows:

Two selfmapping T and S of a metric space X are said to be weakly compatible if they commute at their coincidence points,

i.e., if $Tu = Su$ for some $u \in X$, then $TSu = STu$.

It is easy to see that two compatible maps are weakly compatible.

Definition 11 [18] : Let S and T be two self-mappings of a metric space (X, d). We say that T and S satisfy the $(E.A)$ property if there exists a sequence $\{x_n\}$ such that $\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sx_n = t$ for some $t \in X$.

It is clear from the Jungck's definition [18] that two selfmappings S and T of a metric sapce (X, d) will be noncompatible if there exists at least one sequence $\{X_n\}$ in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$ for some $t \in X$ and $\lim_{n\to\infty} d(Sx_n, TSx_n)$ is either non-zeo or non-existent.

Therefore, two noncompatible selfmappings of a metric space (X, d) satisfy the $(E.A)$ property.

Theorem 2 [18] : Let S and T be two weakly compatible selfmappings of a metric space (X, d) such that

(i) T and S satisfy the property (E.A).

(ii) $d(Tx, Ty) < \max\left\{d(Sx, Sy), \frac{[d(Tx,Sx)+d(Ty,Sy)]}{2}, \frac{[d(Ty,Sx)+d(Tx,Sy)]}{2}\right\} \quad \forall x \neq y \in X.$

(iii) $TX \subset SX$.

If SX or TX is a complete subspace of X, then T and S have a unique common fixed point.

Since two noncompatible self mappings of a metric space (X, d) satisfy the property $(E.A)$, they got the following result:

Theorem 3 [18] : Let A, B, T and S be selfmappings of metric space (X, d) such that

(i) $d(Ax, By) \leq F(\max\{d(Sx, Ty), d(Sx, By), d(Ty, By)\}),\ \forall\ (x, y) \in X^2$,

(ii) (A, S) and (B, T) are weakly compatible,

(iii) (A, S) or (B, T) satisfies the property $(E.A)$,

(iv) $AX \subset TX$ and $BX \subset SX$.

If the range of the one of the mappings, A, B or S is a complete subspace of X, then A, B, T and S have a unique common fixed point.

Definition 12 [**41**]: Let X be a non-empty set and let $d : X \times X \to \mathbb{R}^+$ be a function satisfying the conditions:

(i) $d(x, x) = 0$,

(ii) $d(x, y) = d(y, x) = 0$ implies that $x = y$,

(iii) $d(x, y) = d(y, x)$,

(iv) $d(x, y) \leq d(x, z) + d(z, y)$ for all x, $y, z \in X$.

If d satisfies the conditions from (i) to (iv) then it is called metric on X, if d satisfies conditions (ii) to (iv) then it is called dislocated metric (d-metric) on X and if d satisfies conditions (ii) and (iv) only then it is called dislocated quasi-metric (dq-metric) on X. Clearly, every metric is a dislocated metric but the converse is not necessarily true which can be cleared from the following example.

Example 2 [**41**] : Let $X = \mathbb{R}^+$ define the distance function $d : X \times X \to \mathbb{R}^+$ by $d(x, y) = \max\{x, y\}$ clearly d is dislocated metric but not a metric. Also every metric and dislocated metric is dislocated quasi-metric but the converse is not true which can be cleared from the following example.

Example 3 [**41**]: Let $X = \mathbb{R}^+$. we define the function $d : X \times X \to \mathbb{R}^+$ by $d(x, y) = |x - y| + |x|$ for all $x, y \in X$ evidently d is dq-metric but not a metric nor dislocated metric.

Definition 13 [**41**] : A metric space (X, d) is called complete if every Cauchy sequence in it is convergent to a point in X.

Definition 14 [**41**] : A metric space X is said to be compact if every sequence in it has a convergent subsequence.

Definition 15 [**37**] : In 2011, one important definition was included in the paper of Sintunavarat, W. and Kumam, P. entitled Common Fixed Points for a Pair of Weakly Compatible Maps in Fuzzy Metric Spaces [37].

Let A and S be two self mappings defined on a metric space (X, d). We say that the mappings A and S satisfy $(CLR)_A$-property if there exists a sequence $\{x_n\} \in X$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = Ax$.

Definition 16 [36] : Let X be a nonempty set then a mapping $d_1 : X \times X \to [0, \infty)$ is called a dislocated metric (or simply d_1- metric) if the following conditions hold for any $x, y, z \in X$.

(i) If $d_1(x, y) = 0$, then $x = y$

(ii) $d_1(x, y) = d_1(y, x)$

(iii) $d_1(x, y) \leq d_1(x, z) + d_1(x, y)$

The pair (X, d_1) is called a dislocated metric space (or d-metric space in short). Note that when $x = y$, $d_1(x, y)$ may not be 0.

Definition 17 [21] : Let X be a nonempty set and a mapping $b_d : X \times Y \to [0, \infty)$ is called a b-dislocated metric (or simply b_d-metric) if the following conditions hold for any $x, y, z \in X$ and $s \geq 1$.

(i) If $b_d(x, y) = 0$, then $x = y$

(ii) $b_d(x, y) = b_d(y, x)$

(iii) $b_d(x, y) \leq s[b_d(x, z) + b_d(z, y)]$.

The pair (X, b_d) is called a b-dislocated metric space. And the class of b-dislocated metric space is larger than that of dislocated metric spaces since a b-dislocated metric is a dislocated metric when $s = 1$.

Example 4 [21] : If $X = \mathbb{R}$, then $d(x, y) = |x| + |y|$ defines a dislocated metric on X.

Definition 18 [36] : Let f and g be two self-mappings on a non-empty set X then,

(i) Any point $x \in X$ is said to be fixed point of f if $fx = x$

(ii) Any point $x \in X$ is called coincidence point of f and g if $fx = gx$, and we called $u = fx = gx$ is a point of coincidence of f and g.

(iii) A point $x \in X$ is called common fixed point of f and g if $fx = gx = x$.

Definition 19 [20] : An element $(x, y) \in X \times X$ is called a coupled coincidence point of a mapping $F : X \times X \in X$ and $g : X \to X$ if $F(x, y) = gx$ and $F(y, x) = gy$.

Definition 20 [26] : Two mappings S and T from a metric space (X, d) into itself are called compatible of type (A) if

$$\lim_{n\to\infty} d(STx_n, TTx_n) = 0$$

and

$$\lim_{n\to\infty} d(TSx_n, SSx_n) = 0$$

whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = x \text{ for some } x \in X.$$

Definition 21 [34] : Let ψ be a family of functions $\psi : [0, \infty) \to [0, \infty)$ satisfying the following conditions:

(i) ψ is nondecreasing and

(ii) $\sum_{n=1}^{\infty} \psi^n < \infty$ for each $t > 0$, where ψ^n is the n-th iterate on ψ.

Definition 22 [34] : Let $T : X \to X$ and $\alpha : X \times X \to [0, \infty)$, we say that T is a α-admissible if for all $x, y \in X$, we have $\alpha(x, y) \geq 1 \Rightarrow \alpha(Tx, Ty) \geq 1$.

Hitzler, P. and Seda, A. K. [9] introduced the concept of dislocated metric space (d-metric space). Especially, they introduced the concept of generalized $\alpha - \psi$- contractive type mapping as follows.

Definition 23 [9] : Let (X, d) be a d-metric space and $T : X \to X$ be a given mapping. We say that T is a generalized $\alpha - \psi-$ contractive mapping if there exist two functions $\alpha : X \times X \to (0, \infty)$ and $\psi \in \Psi$ such that for all $x, y \in X$, and we have $\alpha(x, y)d(Tx, Ty) \leq \psi(M(x, y))$ where

$$M(x, y) = \max\left\{d(x, y), \frac{d(x, Tx) + d(y, Ty)}{2}, \frac{d(x, Ty) + d(y, Tx)}{2}\right\}.$$

Definition 24 [39] : Let, A and B be two non-empty set. Then a mapping $T : A \cup B \to A \cup B$ is called cyclic if $T(A) \subseteq B$ and $T(B) \subseteq A$.

Definition 25 [39] : Let (X, d) be a dislocated metric space and U be a subset of X, then

(i) U is called a d-open subset of X if for all $x \in X$ there exists $r > 0$ such that $B_d(x, r) \subseteq U$.

(ii) Also, $B \subseteq X$ is a d-closed subset of X if $(X \setminus B)$ is a d-open subset of X.

Let F be a given map from non empty set X to itself. Then set of all fixed points of F will be denoted by $Fix(F)$ and is given by $Fix(F) = \{x \in X : x = Fx\}$.

Definition 26 [10] : Let $\{a_1\}$ be a sequence in dislocated metric space (A, d). If

(i) $\{a_j\}$ is known as convergent to $a \in A$ as,

$$\lim_{n\to\infty} a_j = 0, \quad \text{if} \quad \lim_{j\to\infty} T_p(a_j, a) = 0$$

(ii) $\{a_j\}$ is known as Cauchy sequence in A, if $\lim_{j,i\to\infty} T_p(a_j, a_i) = 0$.

(iii) Every Cauchy sequence (A, d) is a convergent sequence then that sequence (A, d) is called complete.

Proposition 1 [26] : Let S and T be mappings of compatible type (A) from a metric space (X, d) into itself. Suppose that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = x$ for some $x \in X$.
If S is continuous then $\lim_{n\to\infty} TSx_n = Sx$.

3. Main Theorems

After the establishment of fixed point theorems in dislocated metric space by Hitzeler, P. and Seda, A. K. in 2001, several authors generalized and extended many theorems in dislocated metric space. Some important theorems established in dislocated metric space are presented below in chronological order.

In 2008, Aage, C. T. and Salunke, J. N. established "The results on Fixed Points in Dislocated and Dislocated Quasi-Metric Space [1]". Main aim of them was to establish and discuss a common fixed point theorem in complete dislocated metric space. They established following fixed point theorems in complete d-metric space as follows:

Theorem 4 [1] : Let (X, d) be a complete dislocated metric space. Let $T : X \to X$ be continuous mapping satisfying, $d(Tx, Ty) \leq \alpha d(x, y) + \beta d(x, Tx) + \gamma d(y, Ty) + \frac{\delta d(x,Tx)d(y,Ty)}{d(x,y)} + \mu \frac{d(x,Tx)d(y,Ty)}{d(xy)}$ for all $x, y \in X$ and $\alpha + \beta + \gamma + \delta + 4\mu < 1$ has a unique fixed point.

Theorem 5 [1] : Let (X, d) be a complete dislocated metric space. Let $S, T : X \to X$ be continuous mappings satisfying, $d(Sx, Ty) \leq h \max\{d(x, y), d(x, Sx), d(y, Ty)\}$ for all $x, y \in X$ and $0 < h < 1$ then S and T have unique common fixed point.

In 2010, Rao, K. P. R. and Rangaswamy, P. established the following theorem for four mappings in d-metric space [33]. Main aim of them was to prove several fixed point theorems for weakly compatible selfmappings on a dislocated metric space.

Theorem 6 [33] : Let (X, d) be a complete dislocated metric space. Let $A, B, S, T : X \to X$ be continuous mappings satisfying,

(i) $S(X) \subseteq B(X)$ and $T(X) \subseteq A(X)$ and $T(X)$ or $S(X)$ is a closed subset of X and

(ii) The pairs (S, A) and (T, B) are weakly compatible such that

$$d(Sx, Ty) \leq h \max\left\{d(Ax, By), d(Ax, Sx), d(By, Ty), \frac{d(Ax, Ty) + d(By, Sx)}{2}\right\}$$

for all $x, y \in X$ and $0 < h < 1$ then the mappings A, B, S and T have a common fixed point.

Theorem 7 [33] : Let (X, d) be a complete dislocated metric space. Let $A, B, S, T : X \to X$ be continuous mappings satisfying

(i) $S(X) \subseteq B(X)$ and $T(X) \subseteq A(X)$

(ii) $SA = AS$ and $TB = BT$ and

(iii) $d(Sx, Ty) \leq \phi\left(\max\left\{d(Ax, By), d(Ax, Sx), d(By, Ty), \frac{d(Ax,Sx)d(By,Ty)}{d(Ax,By)}\right\}\right)$

for all $x, y \in X$, where $\phi : \mathbb{R}^+ \to \mathbb{R}^+$ is monotonically non decreasing and $\sum\limits_{n=1}^{\infty} \phi^n(t)$ for all $t > 0$, then

(i) A and S (or) B and T have coincidence point or,

(ii) the pairs (A, S) and (B, T) have a common coincidence point.

In 2012, Jha, K. and Panthi, D. had motivated with the theorems generalized by Kumar, S. in 2008 for weakly compatible mappings in metric space [20] and proved a fixed point theorem for a single pair of weakly compatible mappings in dislocated metric space as follows [14].

Theorem 8 [14] : Let (X, d) be a complete dislocated metric space. Let $S, T : X \to X$ be two continuous self mappings such that

(i) $T(X) \subseteq S(X)$, the pair (S, T) is weakly compatible maps, and

(ii) there exists a number $\alpha < \frac{1}{2}, d(Tx, Ty) \leq \alpha d(Sx, Sy), \ \forall \ \ x, y \in X.$

If one of the subspaces $T(X)$ or $S(X)$ is complete then S and T have a unique common fixed point.

Motivated with the theorem of Vats, R. K.[38], in 2012, Jha, K and Panthi, D. established the following common fixed point theorem for two pairs of weakly compatible mappings in dislocated metric space [13].

Theorem 9 [13] : Let (X, d) be a complete dislocated metric space. Let $A, B, S, T : X \to X$ be continuous mappings satisfying

(i) $T(X) \subset A(X), S(X) \subset B(X)$ and

(ii) The pairs (S, A) and (T, B) are weakly compatible,

(iii)$d(Sx, Ty) \leq \alpha d(Ax, Ty) + \beta d(By, Sx) + \gamma d(Ax, By)$ for all $x, y \in X$ where $\alpha, \beta, \gamma \geq 0; 0 \leq \alpha + \beta + \gamma < \frac{1}{2}$.

Then, A, B, S and T have a unique common fixed point.

In 2012, Panthi, D. and Jha, K. had established the following common fixed point theorem for four mappings in a complete dislocated metric space [27].

Theorem 10 [27] : Let (X, d) be a complete dislocated metric space. Let $A, B, S, T : X \to X$ be continuous mappings satisfying,

(i) $T(X) \subset A(X), S(X) \subset B(X)$

(ii) The pairs (S, A) and (T, B) are weakly compatible and

(iii) $d(Sx, Ty) \leq \alpha[d(Ax, Ty) + d(By, Sx)] + \beta[d(By, Ty) + d(Ax, Sx)] + \gamma d(Ax, By)$ for all $x, y \in X$ where $\alpha, \beta, \gamma \geq 0, 0 \leq (\alpha + \beta + \gamma) < \frac{1}{4}$. Then the mappings A, B, S and T have a unique common fixed point.

In 2012, Jha, K., Rao, K. P. and Panthi, D. had established the following common fixed point theorem for two pairs of weakly compatible mappings in complete dislocated metric space [15].

Theorem 11 [15] : Let (X, d) be a complete dislocated metric space. Let $A, B, S, T : X \to X$ be continuous mappings satisfying,

(i) $A(X) \subset T(X), B(X) \subset S(X)$ and $B(X)$ or $A(X)$ is closed subset of X.

(ii) the pairs (A, S) and (B, T) are weakly compatible and

$$d(Ax, By) \leq \phi \max\left(d(Sx, Ty), d(Ax, Sx), d(By, Ty), \frac{d(Sx, By) + d(Ax, Ty)}{2}\right)$$

for all $x, y \in X$, where $\phi : \mathbb{R}^+ \to \mathbb{R}^+$ is monotonically non-decreasing and $\sum_{n=1}^{\infty} \phi^n(t) < \infty$ for all $t > 0$, then the mappings A, B, S and T have a unique common fixed point.

In 2014, Wadkar, B.R., Bharawadj, R. and Sing, B.S. had established "Common fixed point results in dislocated metric space" [40] which are as follows:

Theorem 12 [40] : Let (X, d) be a complete d-metric space. Let $A, B, S, T : X \to X$ be continuous mapping satisfying,

(i) $T(X) \subset A(X), S(X) \subset B(X)$

(ii) The pairs (S, A) and (T, B) are weakly compatible and

(iii) $d(Sx, Ty) \leq \alpha d(Ax, Tx) + \beta(Ax, By) + \gamma d(Ax, Sx) + \eta d(By, Ty) + \delta d(By, Sx)$ for all $x, y \in X$, where,$\alpha, \beta, \gamma, \eta, \delta \geq 0$ and $0 \leq 2\alpha + \beta + \gamma + \eta + 2\delta$. Then A, B, S and T have a unique common fixed point.

In 2015, Bennani, S., Bourijal, H., Moutawakil, D. E. and Mhanna, S. established "Some New Common Fixed Point Results in a Dislocated Metric Space" [4].

The aim of this paper was to establish several new common fixed point results for four self-mappings in dislocated metric space. The main results of this paper are as follows:

Theorem 13 [4] : Let (X, d) be a d-metric space. let A, B, T and S be four self-mappings of X such that

(i) $TX \subset AX$ and $SX \subset BX$

(ii) The pairs (S, A) and (T, B) are weakly compatible

(iii) $d(Sx, Ty) \leq \alpha d(Ax, Ty) + \beta d(By, Sx) + \gamma d(Ax, By)$ for all $x, y \in X$ where $\alpha, \beta, \gamma, \geq 0$ satisfying $\alpha + \beta + \gamma < \frac{1}{2}$ and

(iv) the range of one of the mappings A, B, S or T is a complete subspace of X.

Then A, B, T and S have a unique common fixed point in X.

Theorem 14 [4] : Let (X, d) be a d-metric space. let A, B, T and S be four self-mappings of X such that

(i) $TX \subset AX and SX \subset BX$

(ii) The pairs (S, A) and (T, B) are weakly compatible

(iii) $d(Sx, Ty) \leq \alpha d(Ax, Ty) + \beta d(By, Sx) + \gamma d(Ax, By)$ for all $x, y \in X$ where $\alpha, \beta, \gamma > 0$ satisfying $\alpha + \beta + \gamma \leq \frac{1}{2}$ and

(iv) The range of one of the mappings A, B, S or T is a complete subspace of X.

Then A, B, T and S have a unique common fixed point in X.
Similarly, in 2015, Bennani, S., Bourijal, H., Mhanna, S. and Moutawakil, D.E. eatablished "Some common Fixed Point Theorems in Dislocated Metric Spaces" [5].
The aim of this paper was also to establish several new common fixed point results for four self-mappings in dislocated metric space. Main results are as follows:
Theorem 15 [5] : Let A, B, T and S be four self-mappings of a d-metric space (X, d) such that

(i) $TX \subset AX$ and $SX \subset BX$.

(ii) The pairs (S, A) and (T, B) are weakly compatible,

(iii) For all $x, y \in X$ and $\alpha, \beta, \gamma \geq 0$ satisfying $\alpha + \beta + \gamma < \frac{1}{4}$. We have, $d(Sx, Ty)[d(Ax, Ty) + d(By, Sx)] + [d(Ax, Sx) + d(By, Ty)] + d(Ax, By)$

(iv) The range of one of the mappings A, B, S or T is a complete subspace of X.

Then A, B, T and S have a unique common fixed point in X.
Theorem 16 [5] : Let A, B, T and S be four self-mappings of a d-metric space (X, d) such that

(i) $TX \subset AX$ and $SX \subset BX$

(ii) The pairs (S, A) and (T, B) are weakly compatible and

(iii) For all $x, y \in X, \alpha, \beta \geq 0$ and $\gamma > 0$ satisfying $\alpha, \beta, \gamma < \frac{1}{4}$, we have

$$d(Sx, Ty) + \alpha[d(Ax, Ty) + d(By, Sx)] + \beta[d(Ax, Sx) + d(By, Ty)] + \gamma d(Ax, By)$$

(iv) The range of one of the mappings A, B, S or T is a complete subspace of X. Then A, B, T and S have a unique common fixed point in X.

In 2015, He, F. established the following "Common Fixed Point Theorem of Four Self Maps in Dislocated Metric Spaces" [8].

Theorem 17 [8] : Let (X, d) be a d-metric space and let A, B, T and S be four self maps on X such that $TX \subset AX$ and $SX \subset BX$. Suppose that there exists a real number with $\lambda \in [0, 1/2)$, satisfying for all $x, y \in X, d(Sx, Ty) \leq M(x, y)$ where,

$$M(x, y) = \max\{d(Ax, Ty), d(By, Sx), d(Ax, Sx), d(By, Ty), 2d(Ax, By)\}.$$

If the range of one of the mappings A, B, S and T is a complete subspace of X, then

(i) B and T have a coincidence point u,

(ii) A and S have a coincidence point v, and

(iii) $Av = Sv = Bu = Tu$.

Moreover, if the pairs (A, S) and (B, T) are weakly compatible, then A, B, S and T have a unique common fixed point y and $d(y, y) = 0$.

In 2015, Zoto, K., Isufati, A. and Kumari, P. S. established some "Fixed Point Results and E. A Property in Dislocated and Dislocated Quasi- metric Spaces" [42].

They proved several fixed points theorems for weakly compatible selfmappings in dislocated and dislocated quasi-metric space, which satisfy E. A Like and common E. A. Like property, satisfying linear type of contractive condition. But, here, it has regarded only the results in dislocated metric space as follows:

Theorem 18 [42] : Let (X, d) be a dislocated metric space and S, T, F and G are self mappings satisfying the conditions:

(i) $d(Sx, Ty) \leq k_1 d(Fx, Gy) + k_2 d(Fx, Sx) + k_3(Gy, Ty) + k_4 d(Fx, Ty) + k_5 d(Gy, Sx)$
For all $x, y \in X$, where the constants k_1, k_2, k_3, k_4 and k_5 are non-negative and $0 \leq k_1 + k_2 + k_3 + k_4 + k_5 < \frac{1}{2}$.

(ii) The pairs (S, F) and (T, G) satisfy common E. A. Like property.

(iii) The pairs (S, F) and (T, G) are weakly compatible Then F, G, S and T have a unique common fixed point in X.

In 2015, Rahman, M. U. and Sarwar, M. established some results on "Fixed Point Theorems for Expanding Mappings in Dislocated Metric Space" [30].

The aim of this paper was to present fixed point theorems in dislocated metric space. They have proved some unique fixed point results for expanding type of continuous self-mapping and surjective expanding self-mapping in dislocated metric space. They first proved some unique fixed point results satisfying expanding condition by taking the continuity of self-mapping and then considering surjective self-mapping in the context of dislocated metric space which are as follows:

Theorem 19 [30] : Let (X, d) be a complete dislocated metric space let $T : X \to X$ be a continuous self-mapping satisfying the condition

$$d(Tx, Ty) \geq a \cdot d(x, y) + b \cdot d(x, Tx) + c \cdot d(y, Ty) \quad \forall \quad x, y \in X$$

and $a > 1, b \in R$ and $c \leq 1$ with $a + b + c > 1$. Then T has a unique fixed point.

Theorem 20 [30] : Let (X, d) be a complete dislocated metric space. $T : X \to X$ be a surjective self-mapping satisfying the condition

$$d(Tx, Ty) \geq a \cdot d(x, y) + b \cdot d(x, Tx) + c \cdot d(y, Ty) \forall x, y \in X$$

and $a > 1$, $b \in \mathbb{R}$ and $c \leq 1$ with $a + b + c > 1$. Then T has a unique fixed point.

Theorem 21 [30] : Let (X, d) be a complete dislocated metric space. $T : X \to X$ be a continuous self-mapping satisfying the condition

$$d(Tx, Ty) \geq a \cdot d(x, y) + b \cdot d(x, Tx) + c \cdot d(y, Ty) + e \cdot d(x, Ty) + f \cdot d(y, Tx) \forall\ x, y \in X$$

with $a + b + c > 1$ and $c \leq 1 + e + f$. Then T has a fixed point.

Theorem 22 [30] : Let (X, d) be a complete dislocated metric space and $T : X \to X$ be a surjective self-mapping satisfying the condition,

$$d(Tx, Ty) \geq a \cdot d(x, y) + b \cdot d(x, Tx) + c \cdot d(y, Ty) + e \cdot d(x, Ty) + f \cdot d(y, Tx) \forall\ x, y \in X$$

with $a + b + c > 1, c \leq 1 + e + f$ and $a, c, f > 0$. Then T has a fixed point.

In 2015, Panthi, D. and Kumari, P.S established "Common Fixed Point Theorems for Mappings of Compatible Type(A) in Dislocated Metric Space" [26]. In this article, they established common fixed point results for two pairs of mappings of compatible type(A) in dislocated metric space. The main results are as follows:

Theorem 23 [26] : Let (X, d) be a complete d-metric space. Let $A, B, S, T : X \to X$ be mappings satisfying the condition

$$T(X) \subset A(X) \ \ \& \ \ S(X) \subset B(X).$$

The pairs (T, B) and (S, A) are compatible of type (A)

$$\begin{aligned} d(Tx, Sy) \quad \leq \quad & \alpha \left[\frac{d(Ay, Sy)d(Bx, Ay)}{d(Ax, Tx) + d(Sy, Ax)}\right] + \beta \left[\frac{d(Tx, Ax)d(Ty, By)}{d(Ax, Tx) + d(Sy, Ax)}\right] \\ & + \left[\frac{d(Ax, Sy)d(Sy, Ay)}{d(Ax, Tx) + d(Sy, Ax)}\right] + \kappa \left[\frac{d(Bx, Ay)d(Tx, Sy)}{d(Bx, Tx) + d(Bx, Sy)}\right] \\ & \delta \left[\frac{d(Bx, Tx)d(Ay, Sy)}{d(Ax, Ty) + d(Bx, Ty)}\right] + \mu \left[\frac{d(Ax, Sx)d(By, Ty)}{d(Bx, Ay) + d(Ax, Ty)}\right] \end{aligned}$$

For all $x, y \in X$ and $\alpha, \beta, \gamma, \delta, \kappa, \mu \geq 0$ such that $0 \leq \alpha + \beta + \gamma + \delta + \kappa + \mu < 1$. If any one of A, B, S, T is continuous then A, B, S and T have a unique common fixed point in X.

Theorem 24 [26] : Let (X, d) be a complete d-metric space. Let $A, B, S, T : X \to X$. Suppose that any one of A, B, S, T is continuous and for some positive integers p, q, r, s which satisfy the following conditions

$$T^s(X) \subset A^p(X) \ \ \& \ \ S^r(X) \subset B^q(X).$$

The pairs (T, B) and (S, A) are compatible of type (A)

$$\begin{aligned} d(T^s x, S^r y) \quad \leq \quad & \alpha \left[\frac{d(A^p y, S^r y)d(B^q x, A^p y)}{d(A^p x, T^s x) + d(S^r y, A^p x)}\right. + \beta \left[\frac{d(T^s x, A^p x)d(T^s y, B^q y)}{d(A^p x, T^s x) + d(S^r y, A^p x)}\right. \\ & +\gamma \left[\frac{d(A^p x, S^r x)d(S^r y, A^p y)}{d(A^p x, T^s x) + d(S^r y, A^p x)}\right. + \delta \left[\frac{d(B^q x, T^s x)d(A^p y, S^r y)}{d(A^p x, T^s y) + d(B^q x, T^s y)}\right. \\ & +\kappa \left[\frac{d(B^q x, A^p y)d(T^s x, S^r y)}{d(B^q x, T^s x) + d(B^q x, S^r y)}\right. + \mu \left[\frac{d(A^p x, S^r x)d(B^q y, T^s y)}{d(B^q x, A^p y) + d(A^p x, T^s y)}\right. \end{aligned}$$

For all $x, y \in X$ and $\alpha, \beta, \delta, \kappa, \mu \geq 0$, such that $0 \leq \alpha + \beta + \gamma + \kappa + \mu < 1$ then A, B, S and T have a unique common fixed point in X.

In 2016, Panthi, D. and Subedi, K. established "Some Common Fixed Point Theorems for Four Mappings in Dislocated Metric Space" [26].

They generalized and extended the results of Amri, A. and Moutawakil, D.[3], Sintu Navarat, W. and Kumam, P. [40] in dislocated metric space. They established some common fixed point theorems for two pairs of weakly compatible mappings with (E. A.) and (CLR) property in dislocated metric space which are as follows:

Theorem 25 [25] : Let (X, d) be a dislocated metric space. Let $A, B, S, T : X \to X$ satisfying thefollowing conditions $A(X) \subseteq S(X)$ and $B(X) \subseteq T(X)$,

$d(Ax, By) \leq k(d(Sy, Ax) + d(Tx, Sy) + d(Tx, Ax) + d(By, Sy) + d(Tx, By), k \in [0, \frac{1}{8}),$

(i) The pairs (A, T) or (B, S) satisfy E. A. property.

(ii) The pairs (A, T) and (B, S) are weakly compatible.

If $T(X)$ is closed then

(i) The maps A and T have a coincidence point.

(ii) The maps B and S have a coincidence point.

(iii) The maps A, B, S and T have a unique common fixed point.

Theorem 26 [25] : Let (X, d) be a dislocated metric space. Let $A, B, S, T : X \to X$ satisfying the following conditions

$A(X) \subseteq S(X)$ and $B(X) \subseteq T(X)$,

$$d(Ax, By) \leq k \max\{d(Sy, Ax), d(Tx, Sy), d(Tx, Ax), d(By, Sy), d(Tx, By), \quad k \in [0, \frac{1}{2}).$$

(i) The pairs (A, T) or (B, S) satisfy E. A. property.

(ii) The pairs (A, T) and (B, S) are weakly compatible.

If $T(X)$ is closed then,

(i) The maps A and T have a coincidence point.

(ii) The maps B and S have a coincidence point.

(iii) The maps A, B, S and T have a unique common fixed point.

Then, they established common fixed point theorems for weakly compatible mappings using (CLR)-property as follow:

Theorem 27 [25] : Let (X, d) be a dislocated metric space. Let $A, B, S, T : X \to X$ satisfying the following conditions $A(X) \subseteq S(X)$ and $B(X) \subseteq T(X)$, $d(Ax, By) \leq kM(x, y), k \in [0, \frac{1}{2})$ where,

$$M(x, y) = d(Tx, Sy) + d(Tx, Ax) + d(By, Sy) + d(Tx, By) + d(Sy, Ax),$$

(i) The pairs (A, T) or (B, S) satisfy CLR-property.

(ii) The pairs (A, T) and (B, S) are weakly compatible.

Then

(i) The maps A and T have a coincidence point.

(ii) The maps B and S have a coincidence point.

(iii) The maps A, B, S and T have a unique common fixed point.

Theorem 28 [25] : Let (X, d) be a dislocated metric space. Let $A, B, S, T : X \to X$ satisfying the following conditions

(i) $A(X) \subseteq S(X)$ and $B(X) \subseteq T(X)$,

(ii) $d(Ax, By) \leq kM(x, y), k \in [0, \frac{1}{2})$.

where

$$M(x, y) = \max\{d(Tx, Sy), d(Tx, Ax), d(By, Sy), d(Tx, By), d(Sy, Ax)\},$$

(i) The pairs (A, T) or (B, S) satisfy CLR-property.

(ii) The pairs (A, T) and (B, S) are weakly compatible.

Then

(i) The maps A and T have a coincidence point.

(ii) The maps B and S have a coincidence point.

(iii) The maps A, B, S and T have a unique common fixed point.

In 2016, Panthi, D. and Kumari, P.S. established "Some Integral Type Fixed Point Theorems in Dislocated Metric Space" [24] which are as follows:

Theorem 29 [24] : Let (X, d) be a dislocated metric space. Let $A, B, S, T : X \to X$ satisfying the following conditions $A(X) \subseteq S(X)$ and $B(X) \subseteq T(X)$

$$\int_0^{d(Ax,By)} \varphi(t)dt \leq k \int_0^{M(xy)} \varphi(t)dt), \quad k \in [0, \frac{1}{2})$$

Where, $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that $\int_0^{\epsilon} \varphi(t)dt > 0$ for each $\epsilon > 0$

$$M(x, y) = \max\{d(Sy, Ax), d(Tx, Sy), d(Tx, Ax), d(By, Sy), d(Tx, By)\},$$

(i) The pairs (A,T) or (B,S) satisfy E. A. property.

ii) The pairs (A,T) and (B,S) are weakly compatible.

If $T(X)$ is closed then

(i) the maps A and T have a coincidence point,

(ii) the maps B and S have a coincidence point,

(iii) the maps A, B, S and T have a unique common fixed point.

In 2017, Rani, A. established "Fixed Point Results for Contractive Type Mappings in Dislocated Metric Spaces" [32]. In this paper it had been presented the following fixed point results for generalized contractive type mappings in dislocated metric space and also had provided some examples to illustrate the results.

Theorem 30 [32] : Let (X,d) be a d-complete metric space. Suppose that $T: X \to X$ is a generalized $\alpha - \psi$-contractive mapping and satisfies the following conditions:

(i) T is α-admissible

(ii) There exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$;

(iii) T is d-continuous .

Then there exists $u \in X$ such that $Tu = u$.

The next theorem does not require continuity of T.

Theorem 31 [32] : Let (X,d) be a complete d-metric space. Suppose that $T: X \to X$ is a generalized $\alpha - \psi$-contractive mapping and the following conditions satisfies:

(i) T is α-admissible

(ii) There exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$;

(iii) If x_0 is a sequence in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n and $x_n \to x$ as $n \to \infty$, then there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $\{x_{n_k}, x\} \geq 1$ for all K. Then there exist $u \in X$ such that $Tu = u$.

In 2017, Wadkar, B. R., Bhardwaj, R. and Singh, B. had established "Some Fixed Point Theorems in Dislocated Metric Space" [39]. This paper deals with the existence and uniqueness of fixed point of a cyclic mapping in dislocated metric space. The main results are as follows:

Theorem 32 [39] : Consider dislocated metric space (X, d) with $m \in N$. Also consider non empty subsets. $A_1, A_2, \cdots, A_m$ of X which are d-closed and $Y = \bigcup_{i=1}^{m} A_i$. Suppose $F : Y \to Y$ is a generalized $\varphi - \psi$ contractive mapping which is cyclic then F has fixed point in $\bigcap_{i=1}^{n} A_i$. Moreover, if for all $x, y \in Fix(F), d(x, y) \geq d(x, x)$ then F has a unique fixed point in $\bigcap_{i=1}^{n} A_i$.

Theorem 33 [39] : Let (X, d) be a complete dislocated metric space with self map F on it. Suppose that there exist $\varphi \in \Phi$, $\psi \in \Psi$ such that for all $x, y \in X$, we have, $\psi(d(Fx, Fy)) \leq \psi(w_d(x, y)) + \varphi(W_d(x, y))$, where

$$\begin{aligned} W_d(x,y) &= \max\left\{d(x,y), d(x,Fx), d(y,Fy), \frac{d(y,Fx)+d(y,Fy)+d(x,Fx)}{6},\right. \\ &\quad \left.\frac{d(x,Fx)d(y,Fy)}{d(x,y)} d(x,y)\right\}. \end{aligned}$$

Then F has a fixed point. Also, F has a unique fixed point if $d(x, y) \geq d(x, x)$ for all $x, y \in Fix(F)$.

Theorem 34 [39] : Let (X, d) be a complete dislocated metric space and $m \in N$. Let d-closed non empty subsets of (X, d) be $A_1, A_2, \cdots, A_m$ and $Y = \bigcup_{i=1}^{m} A_i$. Assume that $F : Y \to Y$ is a ψ- cycle generalized weakly C-contraction. If there exist $x_0 \in Y$ such that $\psi(x_0) \leq \frac{1}{8}$ then F has a fixed point $z \in \bigcap_{i=1}^{n} A_i$.. Also, if $\psi(z) \leq \frac{1}{8}$ then z is unique.

Theorem 35 [39] : Let (X, d) be a complete dislocated metric space, and let $F : X \to X$ be a sub ψ-admissible mapping such that

$$\begin{aligned} d(Fx,Fy) &\leq \beta[\psi(x)d(y,Fx) + \psi(Fx)d(x,Fy) + \psi(F^2x)d(y,Fy) + \psi(F^3x)d(x,Fx) \\ &\quad + \psi(F^4x)\frac{d(x,Fx)d(y,Fy)}{d(x,y)} + \psi(F^5x)\frac{d(x,Fx)d(x,Fy)}{d(y,Fx)} \\ &\quad - \varphi\left\{d(x,Fx), d(y,Fy), \frac{1}{2}[d(x,Fy)+d(y,Fx)]\right\}. \end{aligned}$$

For any $x, y \in X$, where $\varphi \in \Phi, \psi \in \Psi$ then F has a unique fixed point in X.

In 2017, Panthi, D. established "Some Fixed Point Results in Dislocated Metric Space" [23]. In this article, he had established some common fixed point results for a pair of weakly compatible mappings which generalize and extend the results of Amri, A. and Moutawakil, D. E. [3] and Sintunavarat, W. and Kumam, P. [40] in dislocated metric space. The main results are as follows:

Theorem 36 [23] : Let A and S two weakly compatible self Mappings of a dislocated metric space (X, d) such A and S satisfy E. A the property.

$$d(Sx, Sy) < k \max\{d(Ax, Ay), d(Sx, Ax), d(Sy, Ay), d(Sy, Ax), d(Sx, Ay)\}$$

$\forall x, y \in X, k \in [0, \frac{1}{2})$ and $S(X) \subset A(X)$. If $A(X)$ or $S(X)$ is a complete subspace of X, then A and S have a unique common fixed point.

Theorem 37 [23] : Let A and S be two weakly compatible self mappings of a dislocated metric space (X, d) such that A and S satisfy E.A. property,

$$d(Sx, Sy) < k\{d(Ax, Ay) + d(Sx, Ax) + d(Sy, Ay) + d(Sy, Ax) + d(Sx, Ay)\}$$

$\forall x, y \in X, k \in [0, \frac{1}{7})$ and $S(X) \subset A(X)$.

If $A(X)$ or $S(X)$ is a complete subspace of X, then A and S have a unique common fixed point.

Theorem 38 [23] : Let A and S be two weakly compatible self mappings of a dislocated metric space (X, d) such that A and S satisfy $(CLR)_A$ or $(CLR)_S$ property and

$$d(Sx, Sy) < k \max\{d(Ax, Ay), d(Sx, Ax), d(Sy, Ay), d(Sy, Ax), d(Sx, Ay)\}$$

$\forall\ x, y \in X, k \in [0, \frac{1}{2})$, then the mappings A and S have a unique common fixed point.

Theorem 39 [23] : Let A and S be two weakly compatible self mappings of a dislocated metric space (X, d) such that A and S satisfy $(CLR)_A$ or $(CLR)_S$ property and

$$d(Sx, Sy) < k\{d(Ax, Ay) + d(Sx, Ax) + d(Sy, Ay) + d(Sy, Ax) + d(Sx, Ay)\}$$

$\forall\ \ x, y \in X, k \in [0, \frac{1}{7})$ then the mappings A and S have a unique common fixed point.

In 2018, Prudhvi, K. established some results on "Common Fixed Points for Four Self-Mappings in Dislocated Metric Space" [29]. In this paper, author deals with a unique common fixed point theorem for four selfmappings in dislocated metric spaces which generalizes, extends and improves some of the recent results existing in the literature. Main Result of this paper are is follows:

Theorem 40 [29] : Lex Let (X, d) be a complete d-metric space. Suppose S, T, I and $J : X \to X$ are continuous mappings satisfying :

$$d(Sx, Ty) \leq a_1 d(Ix, Jy) + a_2 d(Ix, Sx) + a_3 d(Jy, Ty) + a_4 d(Ix, Ty) + a_5 d(Jy, Sx)$$

for all $x, y \in X$, where $a_i \geq (i = 1, 2, 3, 4, 5), a_1 + a_2 + a_3 + 2a_4 + 2a_5 < 1$.
If $S(X) \subseteq J(X), T(X) \subseteq I(X)$ and if the pairs (S, I) and (T, J) are weakly compatible then S, T, I and J have unique common fixed point.
In 2019, Gaba, H. and Garg, A. K. established "Some Fixed Point Results for Contraction in Dislocated Metric Space" [7]. This paper deals with some fixed point results in dislocated metric space under contraction condition using linear and rational expression in single as well as in couple of mappings for contraction and then they have established some necessary details and some outcomes in dislocated metric space. They have used the abbreviation CDMS for complete dislocated metric space. The main results are as follows:
Theorem 41 [7] : Let (S, d) is a CDMS and : $A \to A$ be a continuous mapping and satisfying the condition:

$$\begin{aligned} d(Tl, Tm) \leq \; & \alpha_1 d(l, m) + \alpha_2[d(l, Tl) + d(m, Tm)] + \alpha_3[d(l, Tm) + d(m, Tl)] \\ & \alpha_4 \left[\frac{d(l, m)d(l, Tm)}{d(l, m) + d(m, Tm)}\right] + \alpha_5 \left[\frac{d(l, Tm)d(m, Tm)}{d(l, m) + d(m, Tm)}\right] \end{aligned}$$

where $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 \geq 0$ with $\alpha_1 + 2\alpha_2 + 4\alpha_3 + \alpha_4 + \alpha_5 < 1$ for all $l, m \in S$, then T has a unique fixed point.
Theorem 42 [7] : Let (S, d) be a CDMS and $T : S \to S$ be a continuous mapping and satisfying the condition:

$$\begin{aligned} d(Tl, Tm) \leq \; & \alpha_1 d(l, m) + \alpha_2[d(l, Tl) + d(m, Tm)] \\ & \left[\frac{d(l, m) + d(m, Tm)}{d(l, Tm)}\right] + \alpha_3[d(l, Tm) + d(m, Tl)] \\ & \left[\frac{d(l, m) + d(m, Tm) + d(l, Tm)}{d(l, Tm)}\right] \end{aligned}$$

where $\alpha_1, \alpha_2, \alpha_3 \geq 0$ with $\alpha_1 + 2\alpha_2 + 8\alpha_3 < 1$ for all $l, m \in S$. Then T has a unique fixed point.
Theorem 43 [7] : Let (X, d) be a CDMS. Let S, T be two self mappings $S, T : X \to X$

(i) $T(X) \subset S(X)$

(ii) S and T are continuous

(iii) $d(Sr,Ts) \leq \alpha_1 d(r,s) + \alpha_2[d(r,Sr) + T_d(s,Ts)]\left[\frac{(d(r,s)+d(s,Ts))}{d(r,Ts)}\right]$
$+ \alpha_3[d(r,Ts) + d(s,Sr)]\left[\frac{\{d(r,s)+d(s,Ts)+d(r,Ts)\}^2}{d(r,Ts)^2}\right]$
where $\alpha_1 + \alpha_2 + \alpha_3 \geq 0$ with $\alpha_1 + 2\alpha_2 + 16\alpha_3 < 1$ for all $x, y \in X$. Then S, T has a unique common fixed point.

4. Conclusion

In 2000, Hitzler, P. and Seda, A. K. first introduced the term dislocated metirc space saying that self distance from a point to itself may not be zero and also included some more definitions, lemmas and theorems, then by Zeyada et. al.(2005) following the definition of Hitzler and Seda. In 2008, Aage, C. T. and Salunke, J. N. established a common fixed point theorem in complete dislocated metric space.

In 2010, Rao, K.P.R. and Rangaswamy, P. established a theorem for four weakly compatible self-mappings in d-metric space. Jha et. al.(2012) (also for single pair weakly compatible mappins), Wadkar et.al.(2014), Bennani et.al.(2015), He (2015), Zoto et.al.(2015) (using E.A. property), Panthi et.al.(2015) (for mappings of compatible type(A), Panthi et.al.(2016) (with (E. A.) and (CLR)-property), Panthi et.al.(2016) (Integral type, for weakly compatible using E.A. property), Panthi (2017) and Prudhvi(2018) established “Some New Common Fixed Point Theorems” for four self-mappings in Dislocated Metric Spaces.

Rahman et.al.(2015) (for expanding mappings), Rani(2017) (for contractive type mappings), Wadkar et.al.(2017) (for cyclic mapping), Gabba et.al.(2019) (under contraction condition) have established some fixed point results in Dislocated Metric Space,

In gist, in the literature which the authors went through, most of the fixed point theorems in dislocated metric space are established for four weakly compatible mappings, some are for single pair of mappings among them some used (E. A.) and (CLR) property and few are in contractive type mapping and integral type.

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